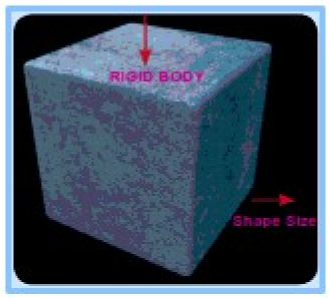
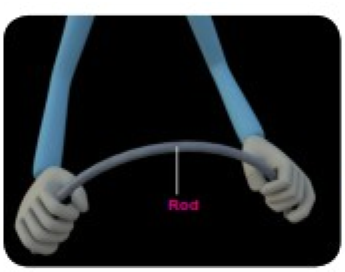
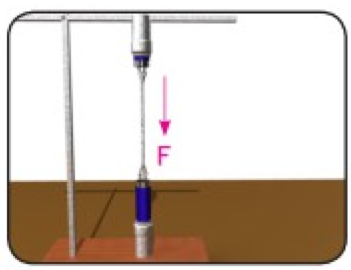
MECHANICAL PROPERTIES OF SOLIDS

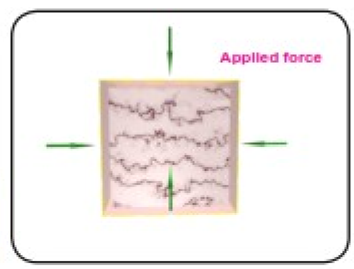
INTRODUCTION

- *A rigid body generally means a hard solid object having a definite shape and size. But in reality, bodies can be stretched, compressed and bent. Even the appreciably rigid steel bar can be deformed when a sufficiently large external force is applied on it. This means that solid bodies are not perfectly rigid.*









A solid has definite shape and size. In order to change the shape or size of a body, a force is required. If you stretch a helical spring by gently pulling its ends, the length of the spring increases slightly. When you leave the ends of the spring, it regains its original size and shape.

Elasticity

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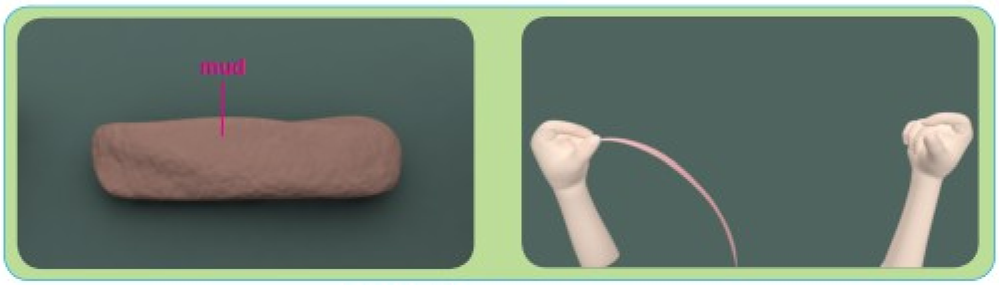
The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as ***elasticity*** *and the deformation caused is known as elastic deformation. However, if you apply force to a lump of putty or mud, they have no gross tendency to regain their previous shape, and they get permanently deformed. Such substances are called plastic and this property is called* ***plasticity****. Putty and mud are close to ideal plastics.*



Plasticity

The elastic behavior of materials plays an important role in engineering design. For example, while designing a building, knowledge of elastic properties of materials like steel, concrete etc. is essential. The same is true in the design of bridges, automobiles, ropeways etc.

ELASTIC BEHAVIOR OF SOLIDS:

- *When a solid is deformed, the atoms or molecules are displaced from their equilibrium positions causing a change in the interatomic or intermolecular distances.*
- *When the deforming force is removed, the interatomic forces tend to drive them back to their original positions.*
- *Thus, the body regains its original shape and size.*

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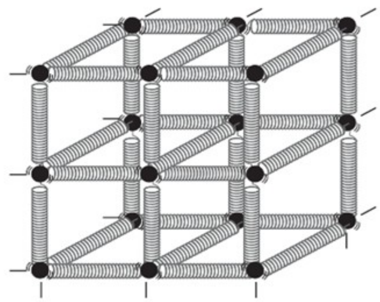


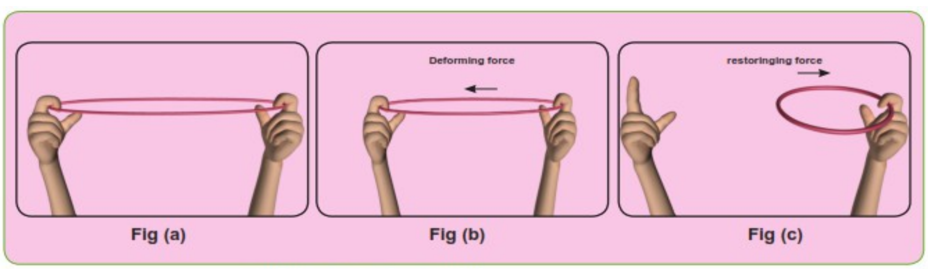
SPRING BALL MODEL

The restoring mechanism is visualized by taking a model of spring ball system. Here, the balls represent atoms and springs represent interatomic forces.

Deformation: *The phenomenon of change in shape or size of a body under the action of force is called deformation.*

Deforming force: *The force,which produce a change in size or shape is called deforming force.*

Restoring force: *Theforce which restores the elongated body to its original shape and size is called the restoring force.*



Elastic behaviour of solids

Now let us studyabout elasticity.

The property of matter by virtue of which an object regains its original shape and size is called elasticity.

Let us perform an experiment to understand the concept.

Stretch a rubber band, what do you observe?

We observe that there is change in shape and size,on releasing rubberband,it comes back to its original length.

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*'The force applied to the rubberband is called '**deforming force'**. Theforce which restores the elongated body to its original shape and size is called the '**restoring force'.***

This property of rubberband, the cause of which rubberband regains original length is called elasticity.

Let us see this example, what do you observe?

When you stretch a piece of chewing gum, and release it, it will not regain its shape and size. This property is known as plasticity.

<u>The difference between elastic and plastic bodies:</u>

Elastic bodies	***Plastic bodies***
1. *1. A body regains its shape when the deforming force is removed.* *2.* 3. *2. With elastic limits, stress is directly proportional to strain.* 4. *3. Hooke's law is obeyed by elastic bodies.* 5. *4. Within elastic limits no permanent set occurs.*	1. *1. A body does not regain its shape and size when deforming force is removed.* 2. *2. Stress and strain are not directly related in plastic bodies.*

6. ***Examples:*** *Quartz, fibre, rubber etc.*	*3. Hooke's law is not obeyed by plastic bodies.* *4. Permanent set occurs.* *3.* 4. ***Examples:*** *Chewing gum, wax, putty etc.*

Stress and Strain:

In the previous concept we studied ***the difference between elastic and plastic bodies****. In this concept we study about stress and strain.*
Let us define stress.
Stress: *The restoring force per unit area, setup inside a body is called stress.*
stress = ==
Unit:*TheSI unit of stress is "newton per metre square" and TheCGS unit of stress is "dyne percentimetre square ".*

Dimensional formula of stress is $ML^{-1}T^{2.}$

Types of stress:*Stress is classified into three types -*

1. *Longitudinal stress.*

2. *Volume or Bulk stress.*

3. *Tangential or shearing stress.*

Normal stress: *If the stress is normal to the surface, it is called normal stress. Stress is always normal in the case of a change in length of a wire or in the case of change in volume of a body.*

1. ***Longitudinal or linear stress:*** *When a normal stress changes the length of a body, then it is called longitudinal stress.*

Or if forces are applied along the length of the body then the stress is known as longitiudinal stress.

Longitudinal stress =

Where F is the deforming force acting on an area 'A' of the body.

Here we can see, cable under tension being stretched by forces acting at its ends.

2. ***Volume or Bulk stress:*** *When a normal stress change the volume of a body, it is called volume stress.*

Or 'if pressure is applied from all the sides, then the stress is reffered to as volume stress.

Volume stress == pressure

Therefore, **Volume stress = *pressure***.

Here we can see the example, when a body (submersible) is immersed in a fluid, the force at any point is normal to the surface of the submersible and the magnitude of the force on any small area is proportional to the area, i.e, the submersible is under the action of a pressure, P.

3. ***Tangential or shearing stress:*** *When the stress is tangential to the surface due to the application of force parallel to the surface, then the stress is called tangential stress.*

Therefore, ***Tangential stress = .***

let us define

Strain: *"The ratio of change in dimension to its original dimension is called strain".*

Strain =

It has no units and dimensions.

<u>Types of strain:</u>

Strain is of three types depending upon the change produced in a body and the stress applied.

1) *longitudinal strain, 2) Volume strain and 3) Shearing Strain.*

1. ***<u>Longitudinal strain:</u>*** *The ratio of change in length to its original length is called longitudinal strain.*

Longitudinal strain = =

:In longitudinal strain, the stress is always perpendicular to the surface of the body.

<u>2.Bulk or Volume strain:</u> *"The ratio of change in volume to its original volume is called bulk strain or volume strain.*

*If **'v'** is the original volume of the body and **'v'**is change in volume, then*

Bulk or Volume strain = .

<u>.Shearing strain or Angular or Tangential strain:</u>

The ratio of the lateral displacement of an upper layer to its distance from the fixed layer is called shearing strain.

*It is denoted by **''.** Consider a block ABCD, See the visual, the face ABCD is fixed and the tangential force F is applied on the upper layer CD, then the block will deform by an angle ABCD be the deformed shape of the block. The angle is called the shearing angle or shearing strain.*

*If **L** is the displacement of the upper surface and **'L'** is the length of the vertical edge. Then*

Shearing strain, =

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Hooke's law:

Experimental study by Hooke revealed that elastic bodies regain their original configuration completely, only up to a limit. He termed this limit as the elastic limit. He found that within the elastic limit, the extension produced in the wire was directly proportional to the load appliedi.e.Stress $\propto$strain

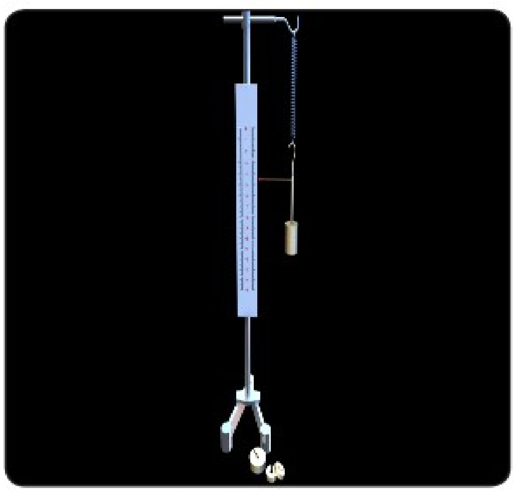
Stress = E(strain).

Where E is constant and is called modulus of elasticity of the material of the body.

Let us perform an experiment to understand Hooke's law.

Experimental verification of Hooke's law:

The apparatus is set as shown. The weights are loaded one by one and unloaded one by one to bring the spring to its elastic mode. Weights are then added in the pan and reading of pointer on the scale is noted. Some more weights are added and the readings are noted once again. The difference between the two gives the extension in the spring due to the weights added in the pan. The procedure is repeated for other weights.



Hooke's law

On plotting a graph between the load and extension, one gets a straight line as shown. Thus, the graph verifies the Hooke's law.

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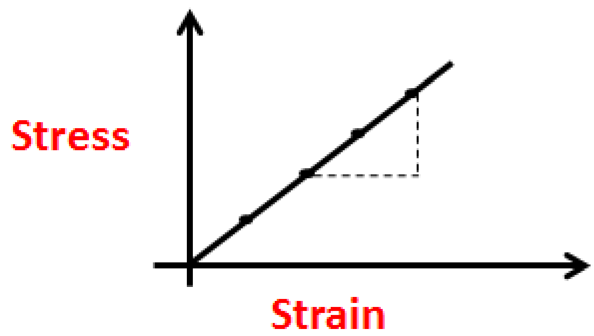
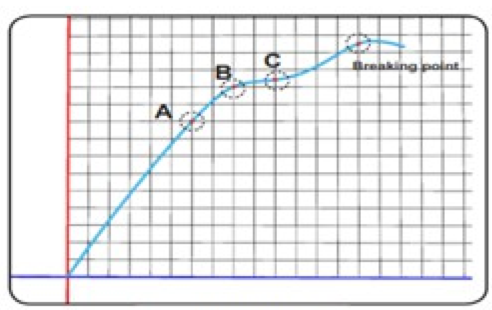


STRESS - STRAIN CURVE:

Steel is more elastic in nature than rubber. To have the same strain for the similar wires of steel and rubber, steel requires more force or load than rubber. So stress is more for steel wire. The elastic nature of a material is calculated by stress only. Hence steel is more elastic in nature than rubber.

Behavior of a wire under the action of gradually increasing load:

When the load is increased in steps, a graph is drawn between stress in Y- axis and corresponding strain on X-axis. Then the curve is obtained. When a wire is clamped at one end and loaded at the other end, it undergoes four following steps:



stress-strain curve

1. Proportional limit:

*In the linear position OA, stress is proportion to strain i.e., Hooke's law is obeyed by the wire up to the point **'A'**, the graph is a straight line. Whenever the stretching force at 'A'is removed, the wire regains its original length. **'A'** is called proportional limit.*

2. Elastic limit:

In the graph 'B' is the elastic limit. Though the wire doesn't obey Hooke's law at 'B' the wire regains its original length after removing the stretching force at 'B'. Up to point 'B' the wire is under elastic behavior.

3. Permanent set (or) yield point:

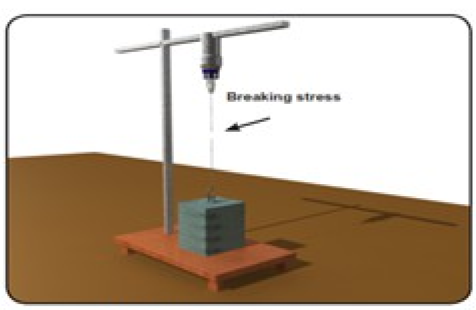
In the graph, 'C' is the yield point. If the stretching force at 'C' is removed, the wire doesn't regain its original length and the length of the wire changes permanently. In this position the wire flows like a viscous liquid. After the point 'C', the wire is under plastic behavior. 'C' is called permanent set or yield point.

4. Breaking point:

When the stress increases the wire becomes thinner and thinner. When the stress increases to a certain limit the wire breaks. The stress at which the wire breaks is called breaking stress and the point 'D' is called "breaking point".

Elastic fatigue:

The state of temporary loss of elastic nature of a body due to continuous strain is called elastic fatigue. When a body is subjected to continuous strain within the elastic limit it appears to have lost elastic property temporarily to some extent and becomes weak.



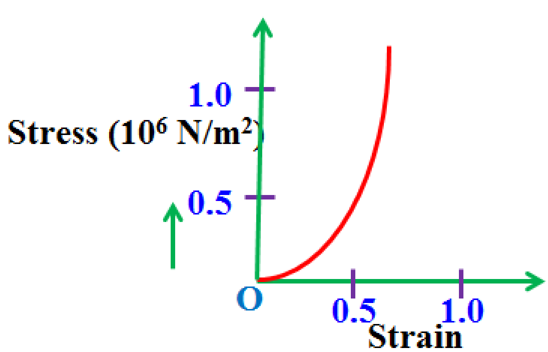
stress-strain

- *If large deformation occurs between the elastic limit and breaking point then the material is ductile.*
- *Ex:Copper, silver, gold etc.*
- *If small deformation occurs between the elastic limit and breaking point then the material is brittle.*
- *Ex:glass, ceramic etc.*

ELASTOMERS:

- *Substances which can be stretched to have large values of strain are called elastomers.*
- *From the graph, it is quite clear that where the elastic region is quite large, the material does not obey Hooke's law.*
- *Stress-strain curve for the elastic tissue of Aorta, which is present in the heart.*

- *Note that although elastic region is very large, the material does not obey Hook's law over most of the region.*
- *Secondly, there is no well-defined plastic region.*



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(Check the animation in slide 4 in ppt)

Substances like tissue of aorta, rubber etc.which can be stretched to cause large strains are called elastomers.

BREAKING STRESS:

- *The breaking stress of a wire is the maximum stress at which the wire breaks.*

Breaking stress=

Breaking stress

a) Depends only on the nature of the material of the wire.

b) Is independent of the length and area of cross- section of the wire.

- *A very long wire suspended vertically may break due to its own weight.*
- *The maximum length of a wire that can be hung without breaking under its own weight is,*

Breaking stress =

The ratio of change in length to the original length is called longitudinal strain

Longitudinal strain =It is defined as the ratio of lateral strain to longitudinal strain.

Poisson's ratio (σ)=

?? = -

Negative sign indicates that the radius or diameter of the wire decreases when it is stretched.

- *Poison's ratio has no units and dimensions as it is the ratio of two strains.*
- *The theoretical limits of Poisson's ratio are from -1 to +0.5*
- *The practical limits of Poisson's ratio are from 0 to 0.5.*

Elastic moduli:

*In the previous concept you studied different types of 'stress and strain'. In this concept you study about **'Moduli of elasticity'**.*

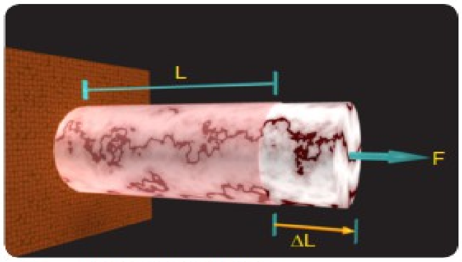
The modulus of elasticity of a body is the magnitude of stress to be applied to produce unit strain.

Corresponding to the three types of strain, there are three moduli of elasticity.

1. *Young's modulus, corresponding to longitudinal strain,*
2. *Bulk modulus, corresponding to volume strain, and*
3. *Rigidity modulus, corresponding to shearing strain.*

Let us study the three types in details:

Young's Modulus:*Within the elastic limit, the ratio of longitudinal stress to longitudinal strain is called Young's modulus. It is denoted by 'Y'*



Young's modulus

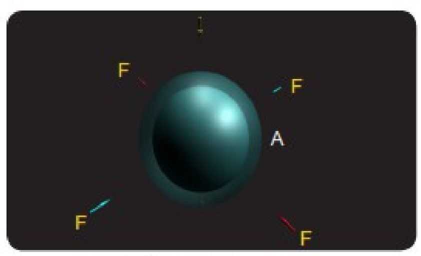
Young's modulus 'Y'= .

Y = = .Where 'F' is the force applied, andL'is the change in length.

Its S.I unit is NM^{-2} or Pascal.

Bulk Modulus:

Within the elastic limit, the ratio of volume stress to volume strain is called Bulk modulus. It is denoted by 'k'.When the volume 'V' of a body changes by under action of a pressure P,



Bulk modulus

Bulk modulus k = ,

k= ,k = .Where 'p' is the stress and is change in volume 'v'. Negative sign shows that volume decrease with increase of pressure.

Rigidity Modulus:

Within the elastic limit, the ratio of shearing stress to shearing strain is called Rigidity modulus. It is denoted by 'η'.

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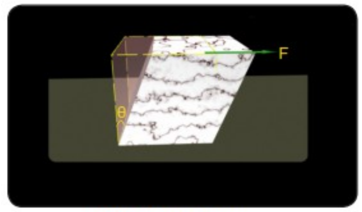


Rigidity modulus

Consider a cube of side L as shown in this video, when a force is applied on its face AHGB for area A fixing in lower face CDEI, the faces perpendicular to the fixed face are displaced by L.

Rigidity modulus (η)=

$\eta =$

$\eta =$

Rigidity modulus is also called shear modulus or torsional modulus.

Solids alone possess rigidity modulus.Liquids and gases do not possess rigidity modlus.

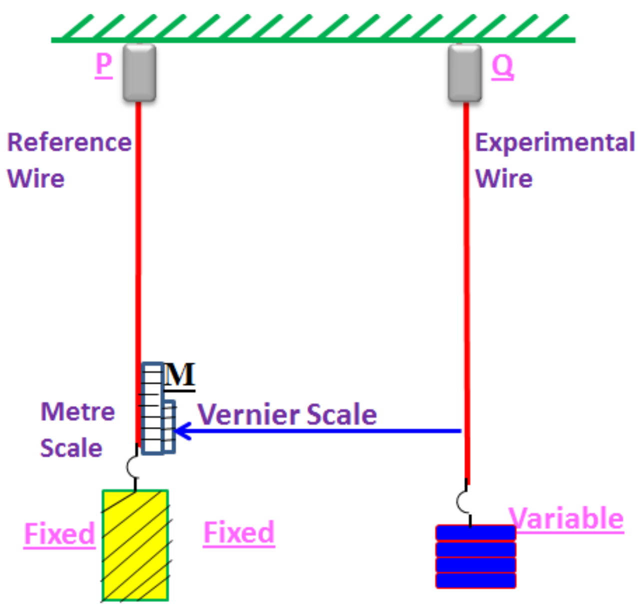
In the previous concept you studied about <u>Hooke's law</u>. In this concept you study Searle's method to determine the Young's modulus of the material of a wire.

<u>Searle's method:</u>

<u>Description:</u> *The apparatus consists of two wires of the same material, length and area of cross section, suspended from a rigid support and at their lower ends two rectangular metal frames. One of the wires is called experimental wire and the other is reference wire. The frame attached to a reference wire carries a constant weight to keep the wire stretched. The frame attached to the experimental wire carries a hanger weight can be slipped as required. A sprit is supported with one end to the frame attached to the reference wire and rests horizontally on the tip of a micrometer.*

Working:

Let us see the working of Searle's apparatus.



(Check the animation in slide 6 in ppt)

1. A suitable load is kept on the hanger then the experimental wire will be straight.

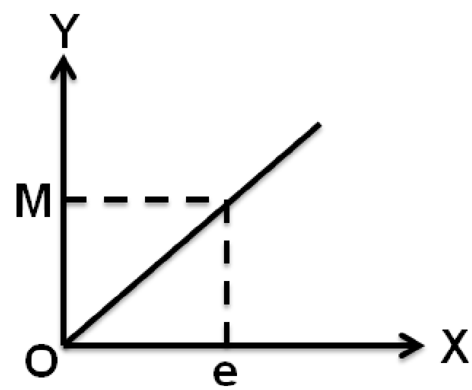
2. Now the micrometer is adjusted so that the air bubble in the spirit level is at its midpoint. The initial micrometer reading is noted.

3. A load of 0.5 kg weight added to the hanger,experimental wire elongates and the frame moves down relative to the other frame. Now the micrometer is adjusted *until the air bubbles in spirit level comes to its initial position and the micrometer reading is noted.*

4. The experiment is repeated at least five times. In every time micrometer readings are noted while increasing and decreasing the load.

5. The difference between the two consecutive readings gives the elongation or extension in the experimental wire when the load is increased by half a kilogram weight.

6. A graph is drawn between the load and extension which is straight line passing through origin.



7. By using a screwguage, radius 'r' of the experimental wire is found. Then length 'L' is measured with the help of ascale.

8. Substituting the values r, L and in the formula.

Young's modulus Y =

(F = mg and A = r^2

Young's modulus of the material is calculated by using the above formula.

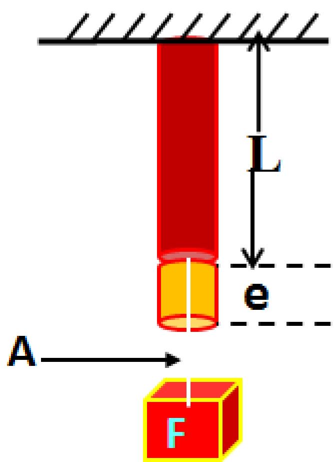
STRAIN ENERGY :

- *When a wire has natural length, the potential energy corresponding to the atomic and molecular forces is minimum.*
- *When the wire is deformed internal forces called restoring forces are setup and work is to be done.*

- *This workdone is stored in the wire as potential energy which is called strain energy.*
- *Strain energy is the energy stored in a body due to its deformation.*

Expression for strain energy:

- *Consider a metal wire of length L and cross-sectional area A fixed at one end and is stretched by an external force F applied at the other end.*



(Check the animation in slide 5 in ppt)

- *The force is so adjusted that the wire is only slowly stretched.*
- *This ensures that at any time during the extension the external force is equal to the tension in the wire.*
- *When the extension is e, the wire is under a longitudinal stress F/A, where 'F' is the tension at that instant of time. The strain is .*
- *Let the force acting on a wire suspended from a rigid support be F. the work done in increasing its length by 'de' is*

$$dW = F.de$$

- *The total work done in increasing the length of wire by 'e' is obtained by integrating the above expression between the limits 'o' (zero) and 'e'.*

W = =

( )

W = This workdone is stored as strain energy in the wire.

Strain energy in the wire =Fe =stretching force extension

Strain energy per unit volume of the wire

=

=

= *stress strain*

When the external force is withdrawn, the stress disappears and the strain appears as heat.

Applications of Elasticity:

Mechanical properties like strength, stiffness (Rigidity), ductility, malleability and brittleness have to be carefully studied to select a material for a particular job.

- *The metallic parts of machines should not be subjected to stress beyond the elastic limit, otherwise they will be deformed.*
- *Beams are the simplest and most common parts of large structures. When beams aresubjected to stress, the different parts are strained in different way. For this purpose, the beam's cross-section is 'I' in shape, where there is advantage of lightness. The flanges are able to withstand the compression and tension force due to loading.*
- *In an arched stone bridge, the stone is compressed and this makes the stone weak. Hence, steel arch is used instead, as steel arch is stronger than the stone arched bridge.*

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- *The thickness of the metallic rope needed to lift a given load is decided using the knowledge of elastic limit of the material of the rope and the factor of safety.*
- *Electric poles are made hollow. A hollow shaft is found to be stronger than a solid one because the torque required to twist a hollow cylinder is greater than the torque required to twist a solid cylinder of same length & radius.*
- *Maximum height of a mountain at any place along its length can be estimated from the elastic behavior of earth.*

It is known that at the base of the mountain, the pressure is 'p' = hrg

where 'h' is height of the mountain, 'r' is density of material of mountain

($3 \times 10^3 kg/m^3$)and 'g' the acceleration due to gravity ($\sim 10 m/s^2$)

The pressure at the base should be less than the elastic limit of the Earth's supporting material ($3 \times 10^8 N/m^2$)

$$h\rho g < 3 \times 10^8 N/m^2$$

$$h < \frac{3 \times 10^8}{3 \times 10^3 \times 10} < 10^4 m$$

Gases being most compressible are least elastic.

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